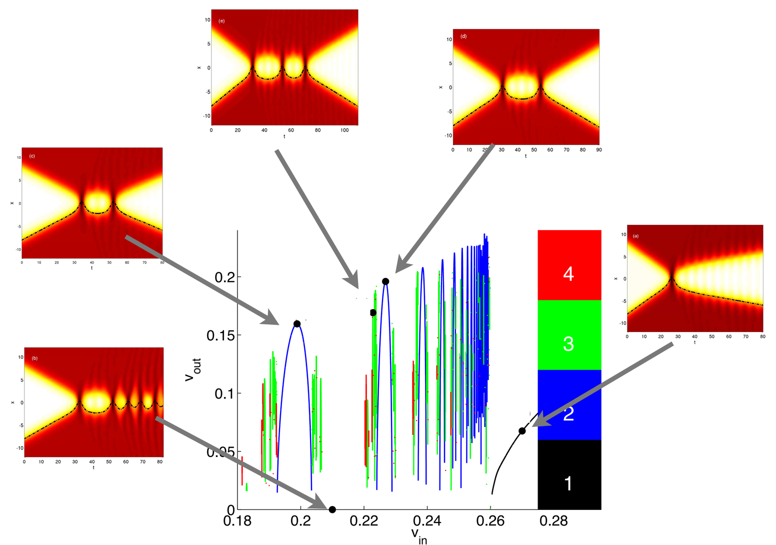
Recent Experiments in the NJIT Applied Math Capstone Lab

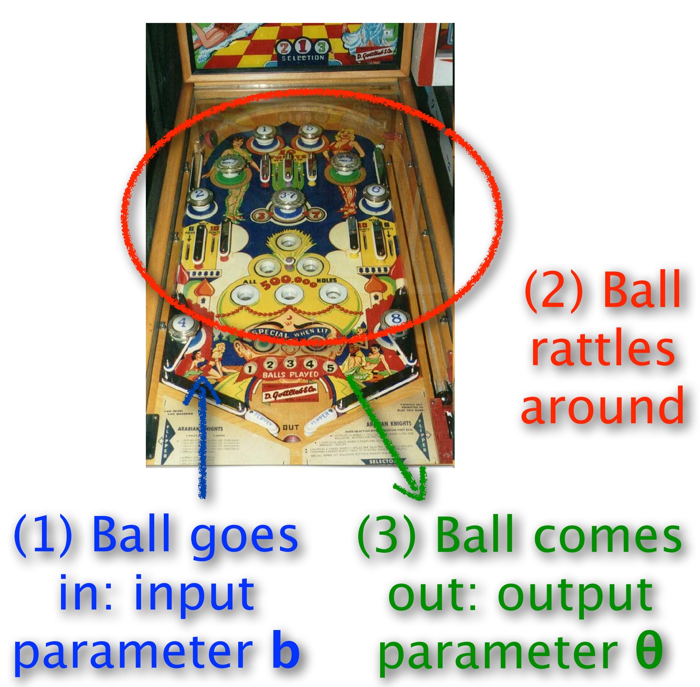
2.5 Problems in Chaotic Scattering

Roy Goodman (professor), Daniel Cargill (TA) and the 2009-2010 Capstone Class

Backstory

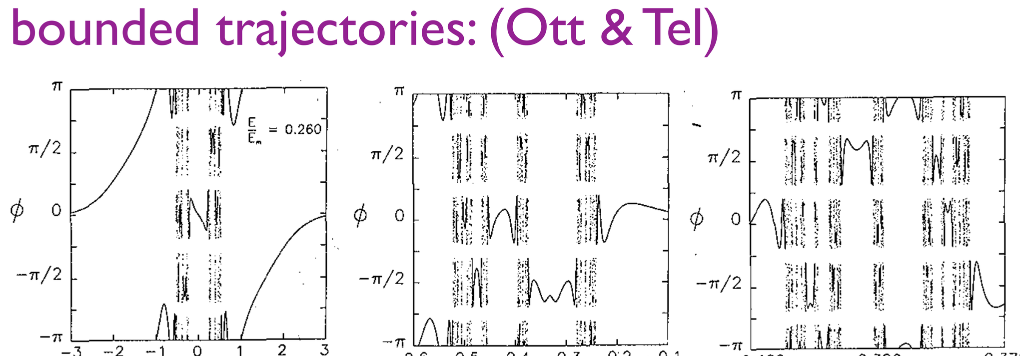


- The first author, the teacher of this class, has spent a good deal of time studying chaotic scattering of solitary waves in non-integrable dispersive nonlinear wave equations, such as the φ^4 equation.
- While he thinks of this phenomenon as "physics," he has only ever seen it in numerical solutions to PDEs, never in an actual experiment, which makes him sad.
- He has an idea: *"This phenomenon is described by a simple system of ODE's. Maybe I can design a sort of table so that a ball rolling on the surface of the table satisfies an ODE system of this type!"*
- He has another thought: *"Can I build such a surface?"*
- He is asked to teach the math department's capstone class. *This is his chance!*
- He devotes the Capstone class to chaotic scattering in this and other systems.
- It works pretty well.



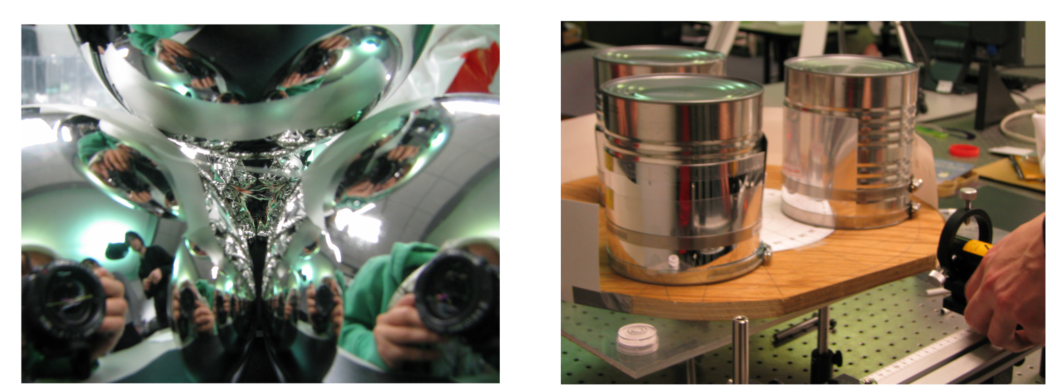
Chaotic Scattering

- Chaotic scattering can occur in open systems, i.e. those in which some trajectories are unbounded.
- A scattering problem refers to one in which a scattering function relates some inputs to some outputs, as in this image from Ott & Tel.
- Chaotic scattering displays sensitive dependence to initial conditions due to transient chaotic dynamics and hyperbolic bounded trajectories: (Ott & Tel)

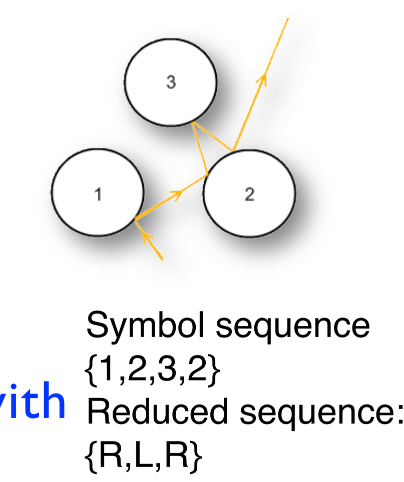


Project 1: Light Scattering from Curved Reflectors

- We considered the scattering of light by curved reflectors in two geometries:
  - Four spheres arranged in a tetrahedron.
  - Three cylinders at the vertices of an equilateral triangle.

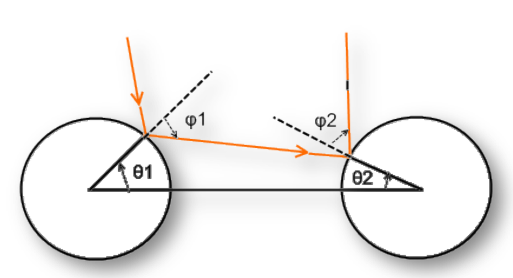


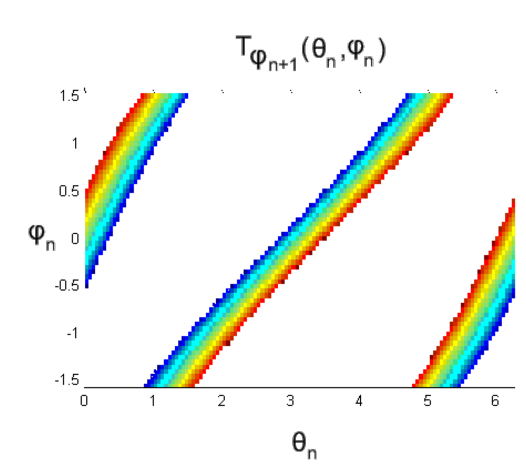
3 Cylinder system:

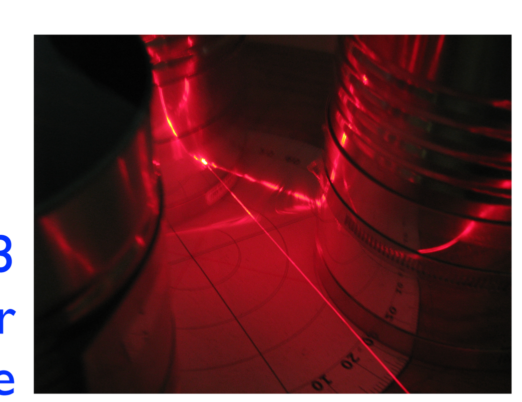


Mathematics learned:

- Symbolic dynamics, shift operators, Markov chains
- Scaling laws associated with chaotic scattering,
- Explicit form of map gives Smale's horseshoe construction (José et al.) and Lyapunov Exponents:







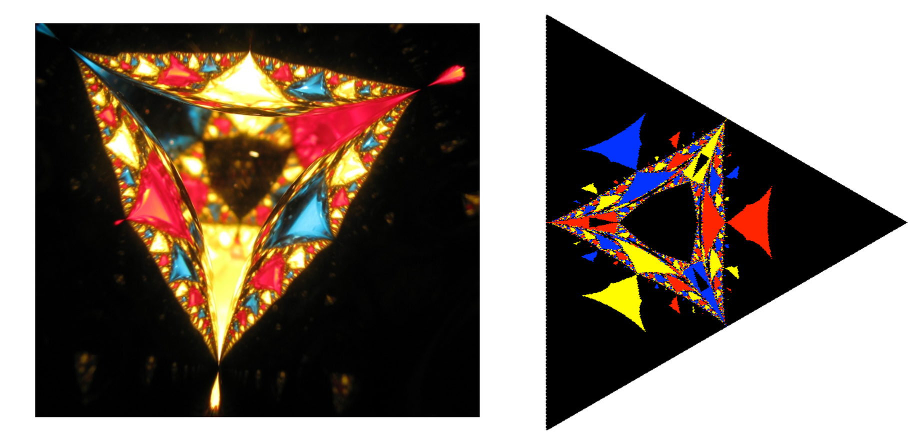
Experiment: a laser, 3 coffee cans, adhesive rear view mirrors, a fog machine and a laser:

Results: Confirm scaling laws between numerics and experiments:

4 Sphere System:

Experiment: 4 cheap Christmas ornaments, colored cellophane, cardboard, a digital camera, and Matlab's image processing toolbox

Photograph & Numerics:



Mathematics learned:

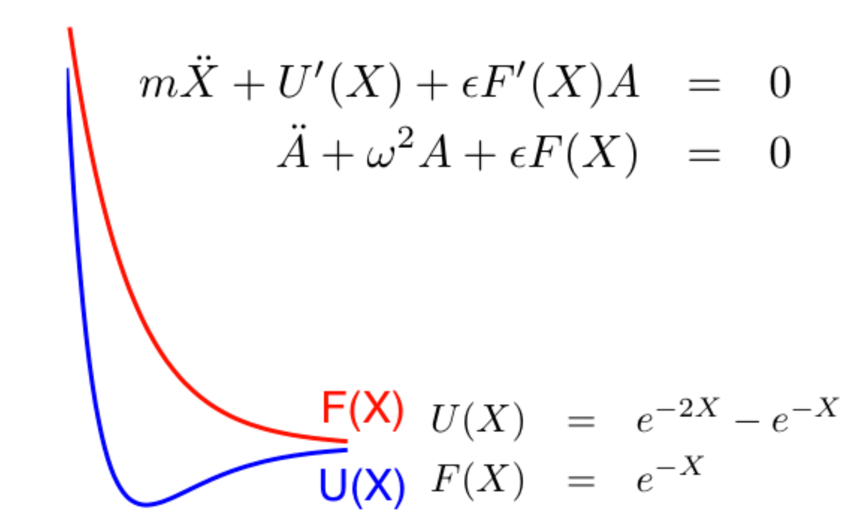
Fractal geometry.

Photo and simulation both give boxcounting dimension 1.5212... (how can it be that good?)

Skewball (a.k.a. Chaos Valley)

Many researchers (Campbell et al, Goodman) have observed chaotic scattering in solitary wave collisions. By formal means, the PDEs governing these interactions may be reduced to Hamiltonian ODE's :

$$m\ddot{X} + U'(X) + \epsilon F'(X)A = 0$$
$$\ddot{A} + \omega^2 A + \epsilon F(X) = 0$$



$$U(X) = e^{-2X} - e^{-X}$$
$$F(X) = e^{-X}$$

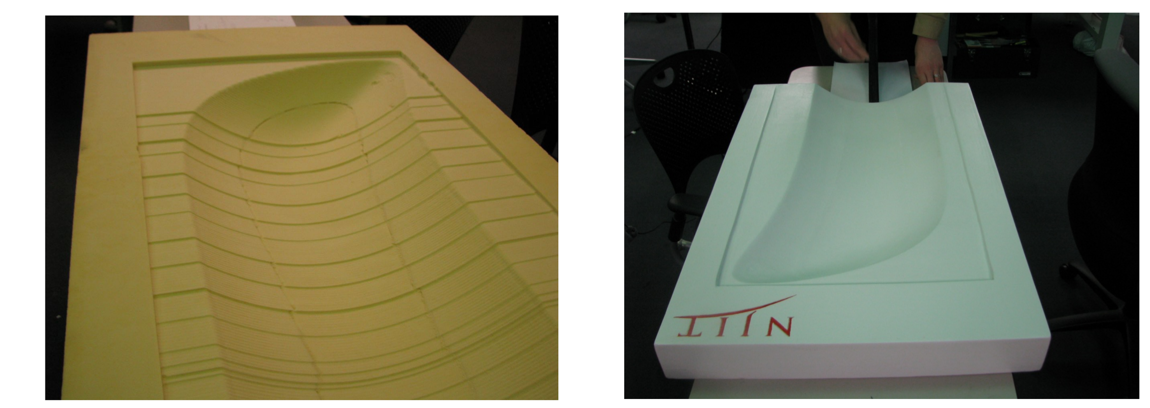
We realized a ball confined to roll on a surface:

$$z = h(x,y) = U(x) + cy^2 + \epsilon y F(x)$$

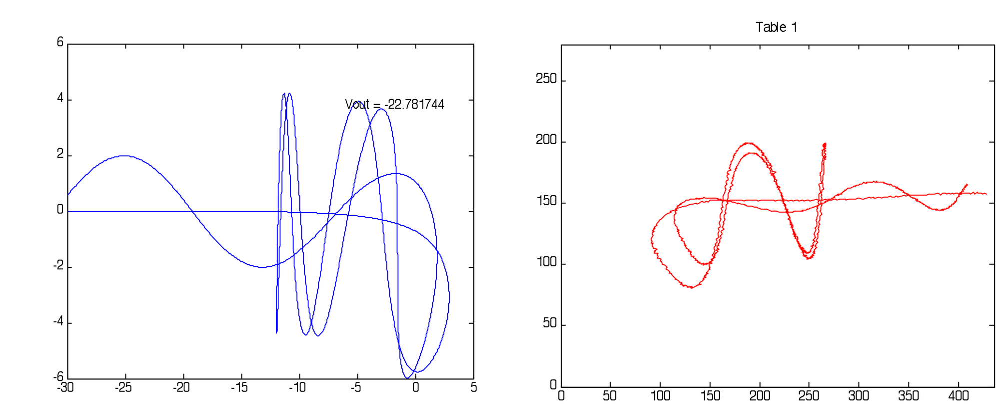
would have very similar dynamics. It evolves under Euler-Lagrange equations:

$$\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix} + \frac{h_{xx}\dot{x}^2 + 2h_{xy}\dot{x}\dot{y} + h_{yy}\dot{y}^2 + g}{1 + h_x^2 + h_y^2}\begin{pmatrix}h_x\\ h_y\end{pmatrix}$$

The surface: carved from high-density urethane foam by a 3-axis mill n the FabLab in the NJIT Architecture Department.



Motion captured with high-speed video, analyzed in Matlab, compared with simulations:



The dynamics can be reduced to a map with a singular Smale horseshoe structure

References

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Campbell et al. Resonance structure in kink-antikink interactions in theory. Phys. D, 9:1–32, 1983.

Goodman. Chaotic scattering in solitary wave interactions: A singular iterated-map description. Chaos (2008) vol. 18, 023113

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Ott and Tél. Chaotic scattering: An introduction. Chaos (1993) vol. 3 pp. 417

Acknowledgments

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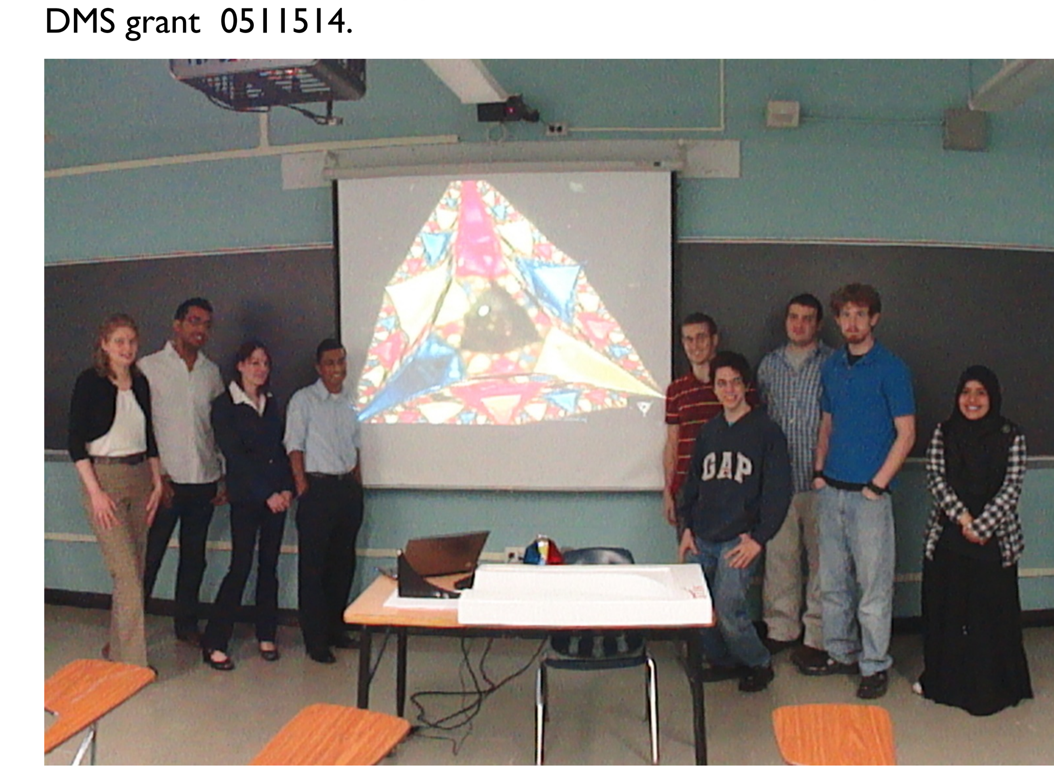

Capstone Students: Catherine Morrison, Sahil Choudhary, Casayndra Basarab, Aminur Rahman, Steven Elliott, Matan Shavit, Michael Bellanich, Matthew Genberg, Fatima Elgammal